

# Horizontal Wells

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## 1 Introduction:

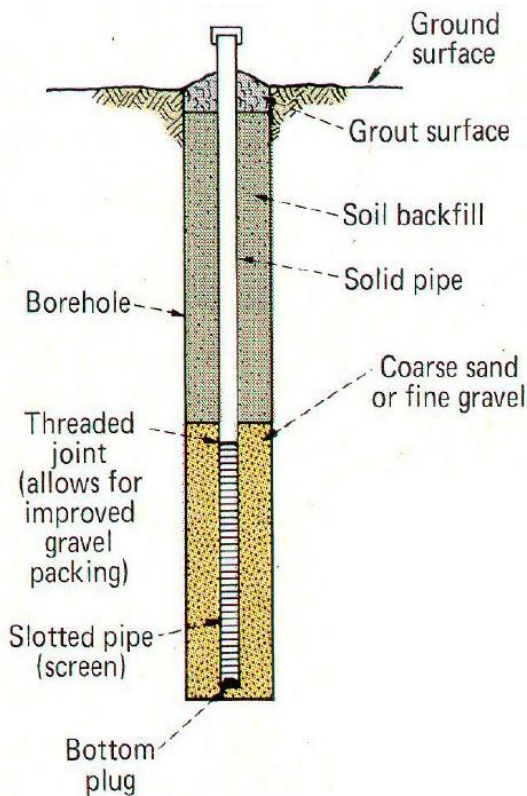
In the environmental community of geologists and engineers there is much agreement about the utility of Horizontal Wells in remedial activities. The knowledge of the design theory is has not caught up with practice. In this brief paper we will go through simple design features and programs which will enable the practitioner to design effective collection and distribution wells for both compressible and non-compressible flows—for both air and water systems. The orientation of a “horizontal well”, lying within the plane of the aquifer, provides a greater and more efficient method to collect or distribute fluids or gasses to or from the formation on a uniform basis.

## 2 Water and non-compressible fluids

Water and many liquids are non-compressible. That fact simplifies the calculations for head losses. Horizontal wells were initially used to collect groundwater into supply galleries or large openings where the water-bearing formations were relatively thin. With all wells, the quantity of water flowing into a well depends upon the hydraulic gradient and the permeability of the formation. In geologists' terms, one would use the word “drawdown” to express the difference between a static water level and the water level in a well being pumped. This difference in head is the driving force which brings water into the well. When one has a relatively thin aquifer, there is only so much drawdown permitted—ie the permeability of the aquifer—which controls the quantity of water available.

The drawdown is also limited by the size of the openings on the well screen, and the viscosity of the liquid being pumped. Typical vertical wells are shown below.

**Figure 1: Typical Vertical Well**



In the figure, a typical vertical well is represented, where the grout plug helps seal the well from the surface water intrusion. The vertical well is constrained by the thickness and permeability of the aquifer. Because the horizontal well is constructed in the length of the aquifer rather than its cross section the horizontal wells are substantially more efficient.

The development of horizontal wells technology and horizontal well theory and analysis has been developed from the petroleum industry. The analysis of a horizontal well's capacity is quite complex and beyond the scope of this article, but analytical information can be obtained from books on reservoir engineering and websites such as [www.Petrowiki.com](http://www.Petrowiki.com).

Figures 2 and Figure 3, Illustrate a typical hydraulic gradient profile for a vertical well. The cone of depression is generally circular around the well. The flow from an aquifer is  $Q$  (flow) is equal to  $k$  (hydraulic conductivity or intrinsic permeability) times the change in hydraulic head with respect to the distance from the well, and the area. This is a calculated value obtained by pumping from the reservoir. According to the USGS Publication 708,  $K$ , the hydraulic conductivity is

measured in terms of feet or meters per day, and is defined as the unit length of time required to transmit a unit volume of liquid at the prevailing dynamic viscosity through a unit cross sectional area under a unit change in hydraulic head through a unit length of flow<sup>iii</sup>.

The general formulation for flow into a well from an aquifer is given by Darcy, as:

$$q = (A/L)(k/\mu)\Delta p \quad \text{Equation (1)}$$

Where  $A$  is the area (total open area of all the openings used to collect or disperse fluid),  $L$  is length,  $k$  is the hydraulic conductivity -usually a constant for a reservoir,  $\mu$  is the viscosity of the fluid<sup>iii</sup> and  $\Delta p$  is the change of pressure (hydraulic head).

The figure below shows a simplified explanation of the shape of the drawdown curve in a vertical water well from an unconfined aquifer. For other fluids in similar formations, the principles are the same. (For oils, the viscosity of the liquid will be greater and the radius of influence will be substantially smaller.) Notice that the cone of depression is curved and asymptotically approaches the static ground water level. As the distance from the well increases, the velocity of the water into the well is decreased to the point where it is effectively zero. This defines the practical area of capture effectiveness for a well. For a horizontal well there is a difference in configuration and thus a difference in the shape of the drawdown curve. The drawdown curve is theoretically linear

over the length of the collection pipe, and ovoid at the end at the first hole in the horizontal well and at the last hole in the well.

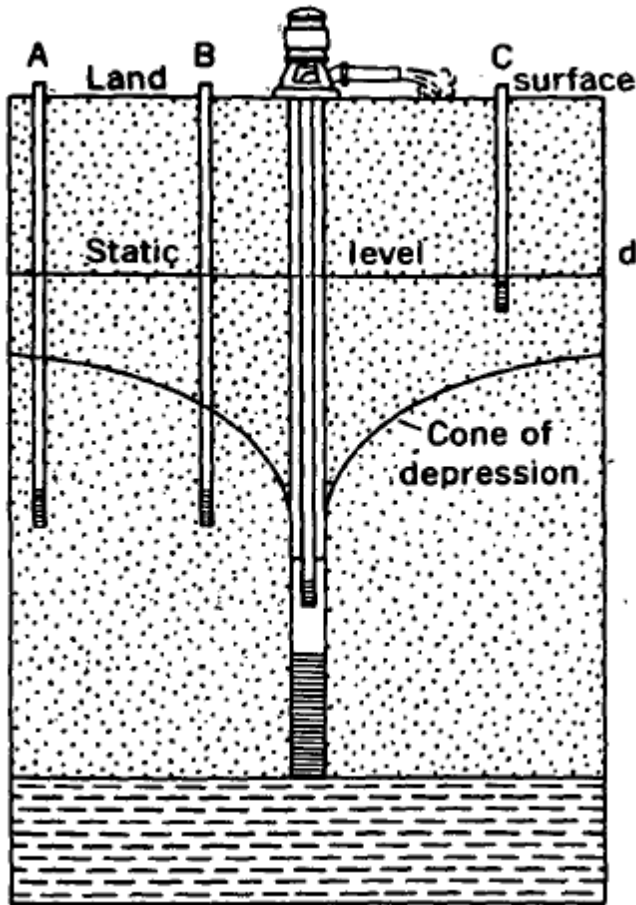
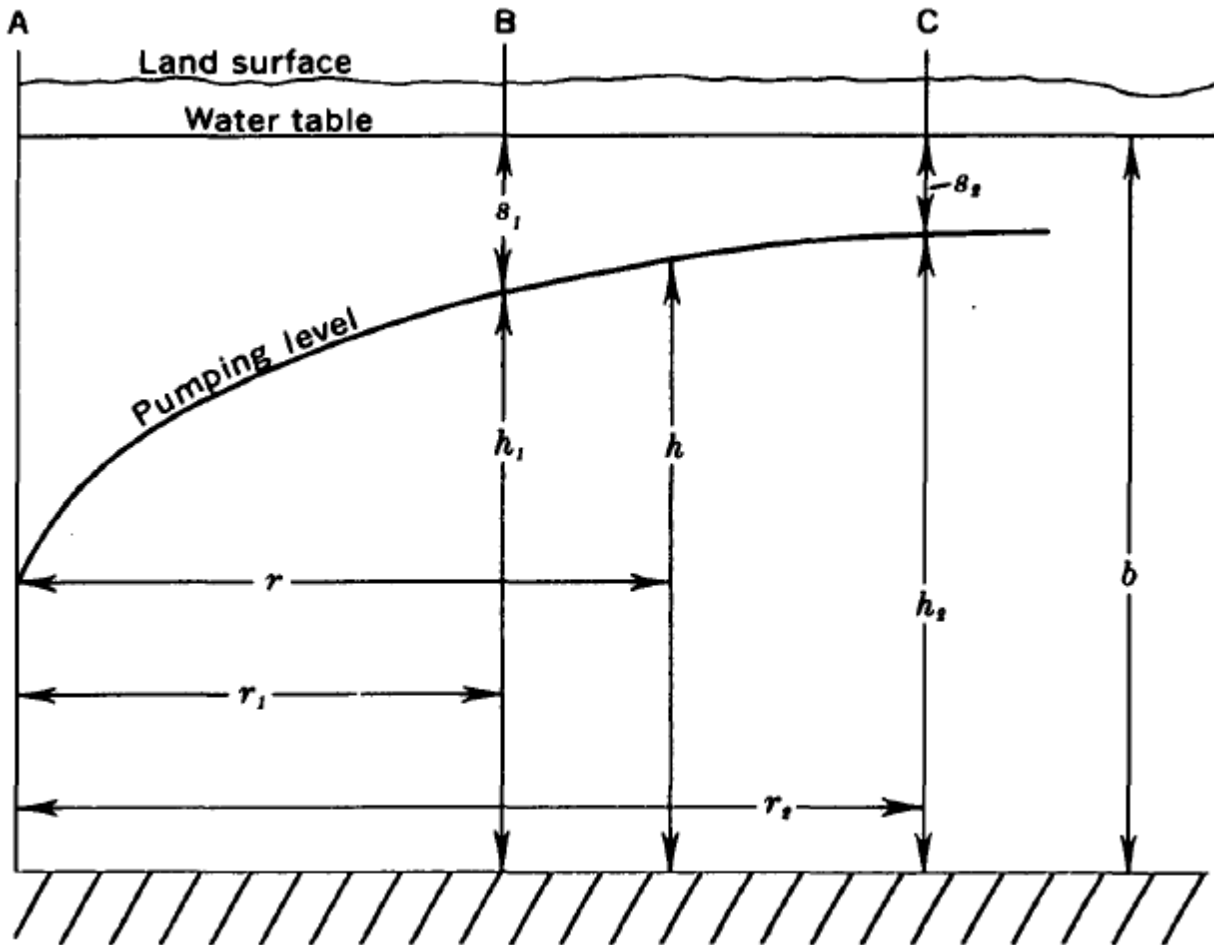


Figure 2: Illustration of a pumping well cone of depression in an unconfined aquifer. The effective water levels in wells A and B have declined but they are still usable. Well C, however is dry because the groundwater pumping from the larger and deeper well has lowered the static water level in well C. (Drawing Source: USGS Pub. 708: Groundwater Hydraulics)

Figure 2 illustrates a half cross-section of the cone of depression around a well in an unconfined aquifer:



**Figure 3: Cone of depression (liquid surface level) from a pumping well in an unconfined aquifer. The distances  $r$  through  $r_2$  are radial from the centerline of the well, and the heights  $h$  through  $h_2$  are the available head at the distance when the well is pumping. The height  $b$  is the maximum available head.**

Equation 2 illustrates the relationship between pumped flow, and the available head for an unconfined aquifer.

$$\log_e \frac{r_2}{r_1} = \frac{2\pi k}{Q} \left[ \frac{h_2^2 - h_1^2}{2} \right] \quad (\text{Equation 2})$$

In equation 2, the variables  $r$  and  $h$  are related to Figure 2.  $Q$  is the pumping rate, and  $k$  is defined above as the hydraulic conductivity of the aquifer. For confined aquifers, the same general formula is applied but the boundaries of the aquifer and the pressure of the aquifer limit the available head ( $h$ ).

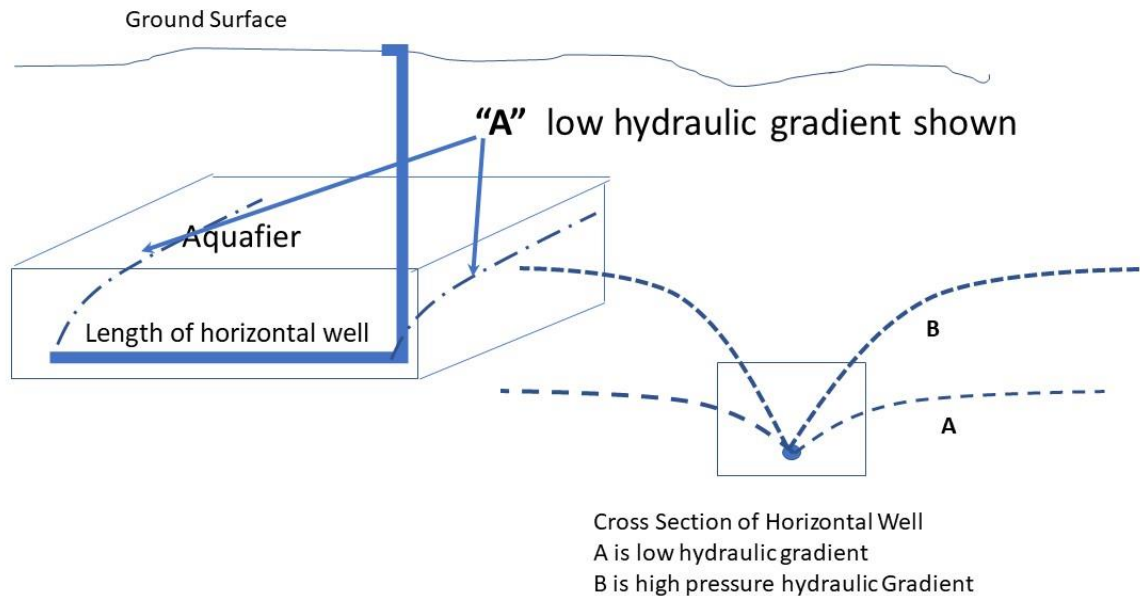
Figure 4 represents the hydraulic regimen in a horizontal well in a confined aquifer.<sup>iv</sup> In the diagram, the A curve is representative of an aquifer under either natural or very low- pressure conditions. The B curve illustrates the hydraulic gradient profile of an aquifer under higher pressure.

For horizontal wells, the formula is modified both in length of the well. At the end of the well, and the connection to the riser pipe, the flow is often ovoid or semicircular. Depending upon the viscosity of the fluid and the porosity and conductivity of the aquifer, several patters for flow into the well can emerge. Immediately around the well, there may be a head decrease which does not stretch to the boundaries of the aquifer. Local effects may give false high readings for aquifer performance. In time, however, as the aquifer drains, the hydraulic pressure profile will flatten out and approach a steady state figure. The area of drainage from the aquifer will increase with time. It is important to develop characteristics for the aquifer using pumping tests and other measures before installing a horizontal well. The cost of a horizontal well may be significantly higher than that for a vertical well, depending upon the size (diameter) and depth of the well and the length in the aquifer. In the oil fields it is often customary to install a solid pipe in the aquifer and then use high pressure water to drill holes in the pipe to create the required number of holes.

In formations with fine soil particles, such as soft sandstone, sands, and silts, and especially when the piping is used for extraction, it is not unusual for the sand and small particles near the holes in the pipe to move into the pipe, and form a series of small external pockets around each of the holes. This is normal and should not adversely affect the performance of the pipe by accumulating solids from the aquifer.

With a horizontal well, the length of the withdrawal zone is very great with respect to the thickness of the aquifer. The flow into the horizontal well is still limited by the thickness of the aquifer and available head, porosity of the medium, viscosity, etc. For a horizontal well, the well has a greater length in the aquifer and is equivalent to pumping from a very, very, thick aquifer.

**Figure 4 Illustration of Horizontal Well Hydraulics**



### 3 Remedial activities using Horizontal Wells

In vertical wells, the hydraulics are constrained by the length of the well screen, the size of the openings, the diameter of the well, the thickness of the aquifer, and the viscosity of the material flowing into the well.

Horizontal wells can be designed to have less open area per length of pipe than conventional vertical wells. In a vertical well, screening is most often installed for the purpose of excluding formation particles from entering the well. In horizontal wells, the individual openings are larger, but the total open area can be designed to be less than well screens. Each of the openings on a horizontal well relatively small, and the flow around each opening may be higher and create small localized voids around each of the holes.

Johnson Well Screen<sup>v</sup>, is an example of one of several manufacturers of well screens. Some of the screens are made from PVC, stainless steel, and bronze. Typical open areas for a one foot long (0.3048 M) well screen on a 6" (15.24 cm) pipe range from 6.2 sq.in (40.65 sq.cm) to 36 sq.in (232.26 sq.cm). By comparison, a 6" pipe with 12- 0.5" diameter holes drilled per foot of pipe would have an open area of 2.536 sq. in. (16.361 sq.cm). The greater open area means that fluid can pass through the well screen with low hydraulic losses<sup>vi</sup>.

In remediation, the two most common problem arise in the areas of vacuum extraction and vapor stripping. Vacuum extraction is where one pulls a vacuum on the formation to withdraw, gases, vapors and fluids, and vapor stripping where air is blown through the contaminated zone in an effort to encourage the contaminants to sublime or evaporate.

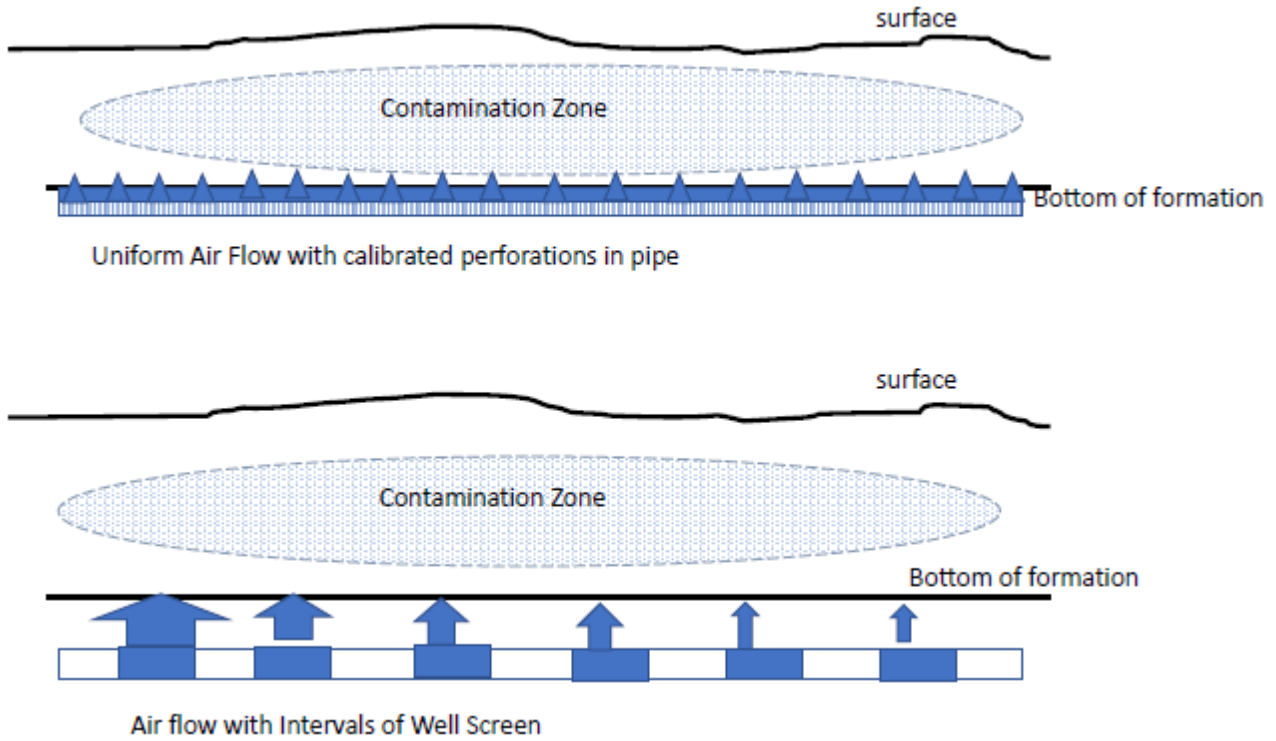
When vapor stripping is used as a remediation technique, it is often used in conjunction with vapor extraction so that volatile contaminants removed by blowing air beneath the contaminated zone, and are suctioned from above the contaminated zone. Horizontal wells are ideally suited for this activity because of the ability to direct the flow in or out into areas where it is needed and tailor the quantity of fluid injected or removed to the needs of the contamination zone for remediation.

An incomplete understanding of the nature of hydraulics and how to collect or distribute flow along a pipe led to the following failure. An acquaintance designed a horizontal air sparging well which contained over 150 feet of well screen. The well screen open area was so great that air was distributed into the formation in the first 20 feet of screen, and the remaining screen was an entire waste of resources. Similarly, while a drain system with the same type of configuration might be more effective, it would be highly dependent upon the formation permeability. In effect, it could serve just as well as a large horizontal hole where the first 20 feet or so of the screen would be the only effective portion.

By comparison, a properly designed horizontal well collection vacuum consisting of 12-400 foot (122 meters) long was installed successfully in a large contamination zone, and in conjunction with vertical sparge wells, collected over 100 tons of chloroform and carbon tetrachloride from a spill at a Louisiana chemical works in less than a year.

The figures below will serve to illustrate the benefits of horizontal wells. If the well screens are spaced along the length of the well at periodic intervals (indicated by dark blue rectangles), the flow will not be uniform, nor directed to the area where is. The horizontal well with calibrated perforations is show in the top half of the drawing. The air flows are represented by the arrows. The air flow in the horizontal well which used well screens is significantly greater at one end than the other. This leads to incomplete remediation. In the top figure the perforated pipe flow is uniform and delivers air to the entire formation uniformly, resulting in better and more uniform remediation.

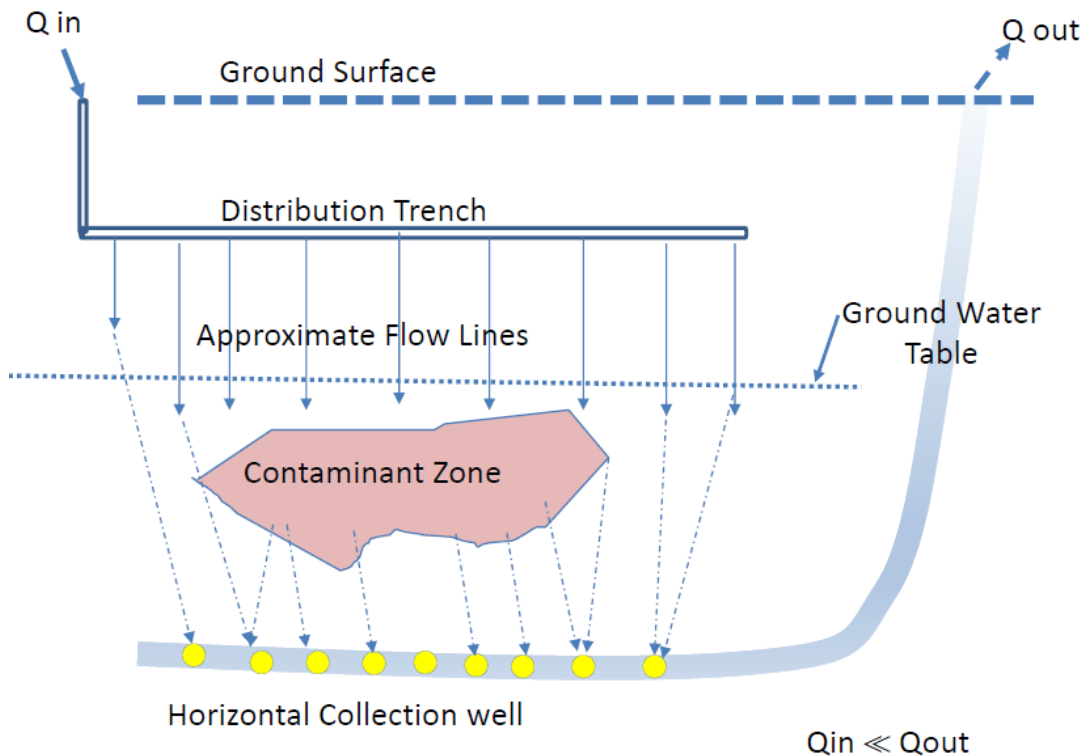
Comparison of air flow through calibrated perforated pipe versus placed screen intervals in a horizontal well



**Figure 5: Comparison of fluids flows using horizontal wells and screened wells. Flow is from left to right.**



## Horizontal Well & Distribution Trench for subsurface cleanup



**Figure 6 Cleanup of a contaminated zone using horizontal wells**

The idea of uniform injection and uniform withdrawal is applicable depending upon the particular application. The most effective type of distribution system is one which is tailored to the piping system<sup>vii</sup>.

### 4 Orifices, Pipe Losses, and how to tailor the pipe openings to meet the required flows.

The formula for energy losses through a nozzle or orifice plate is the same, just the coefficient of discharge is different. That formula is shown in Equation 3:

$$Q = C_d \cdot A_o \cdot [2gh]^{0.5} \quad \text{Equation (3)}$$

**Q** is the flow, **C<sub>d</sub>** is the discharge coefficient, **A<sub>o</sub>** is the area of the orifice and **2gh** is the acceleration of gravity times the hydraulic head. All of it has to be in the same appropriate units. The discharge coefficient is dimensionless and is between 0.59 and 0.61 for most orifice plates, depending upon Reynolds Number for the fluid system<sup>viii</sup>.

Michael Duchene and Edward A McBean reported the head loss coefficients on a number of different piping systems as equal to 0.6 to 0.66 depending upon the depth of burial and the depth of flow in the distribution piping. They principally experimented on piping systems which had four openings around the pipe<sup>x</sup>. They also accounted for differences in hydraulic head on the various openings due to their location on the pipe.

The challenge in piping distribution system design is to get the head losses and the flow through the orifice holes equal to the flow in the pipe. This can require a bit of ingenuity in the design process. The ideal situation is to design the distribution system so that the flow and the losses at the end of the pipe are as close to zero as practical.

One method of way of designing the distribution system would be to analyze the pipe at each perforation for the flow recalculating the flow in the pipe every time the flow is reduced by a flow through an orifice **q**. This gives a flow of **Q** at the first orifice, and **Q-q** between the first and second orifices, and so on. The reduced flow provides a new Velocity  $V_{(Q-q)}$ , and a new set of friction losses for the next length of pipe between the perforations. This would require n+1 different frictional calculations for head loss and friction in the length of the perforate pipe. That calculation is rigorous but time consuming and cumbersome.

An old article in a text book from the 1960's provides a good answer<sup>x</sup>. Fair and Geyer suggested that with a pipe of constant diameter, the head losses through the nozzle were approximately equal to the friction losses equal to about 1/3 the frictional flow in the pipe.

Put in scientific terms, the friction head or **H<sub>f</sub>** is equal to the value shown in Equation 4:

$$H_f = (KQ_o^2/L^2) * (l-l^2/L + \beta/L^2) \quad \text{Equation .4}$$

Where **K** is the hydraulic coefficient equivalent to the head losses in the total length of the pipe at full flow conditions; **L** is the length of the pipe, **l** is the fractional length of the pipe where the losses are occurring, and **Q<sub>o</sub>** is the total flow in the pipe at maximum conditions. Remember that you will need to have the head greater than the hydraulic head on the outside of the pipe.

Another way to look at the solution to the problem is to perform an analysis across the orifice. In a simple case we have  $q_n = C * A_o^2 * [2gh_d]^{0.5}$  (Equation 3). If the value of C is 0.60, and the value of the large pipe flow is **Q** and the individual orifice value is **q<sub>n</sub>**, and the hydraulic head differential between the inside of the pipe and immediately outside the pipe is **h<sub>d</sub>** which accounts for the external pressure due to the submergence of the pipe. If one set the nozzle losses equal to or greater than the pressure head against the pipe plus the pipe friction, the design works.

An example will help. Given a 3" diameter pipe 300 feet long. I want to distribute 100 gallons per minute through it uniformly. Head losses through the length of the pipe are 4.47 feet of head loss per 100 feet of pipe. The pipe is submerged by 3 feet of water. The total head loss is then  $4.47 * 3 + 3 = 16.41$  feet of head or just about 5 PSIG. (Note: 2.31' of water submergence  $\approx$  1 psig, or 6.9 kilopascals)

If you want to use 30 nozzles or orifices, each one should take about 3.33 gpm.

The nozzle size should be  $q = 0.60 A^2 [2gh]^{0.5}$ . In the proper units  $q = 7.42 * 10^{-3}$  Cubic feet per second,  $g = 32.18$  ft/ second<sup>2</sup> and  $h = 16.41$  ft. Running the numbers that gives **A<sup>2</sup>** as  $5.381 * 10^{-4}$  Square feet or 0.0775 square inches. That translates to a hole of approximately 0.0987 inches

in diameter or a 3/32 inch diameter hole. The water will flow out uniformly. The water will also flow in uniformly.

## 5 Compressible Flow

Compressible flow occurs in air and other gases. The formula must account for a few more variables such as the initial and final states and temperatures. As the air crosses the orifice it changes temperature, decreasing sharply as it expands from the nozzle. In order to prevent the formation from plugging into an ice flow, one must account for the change in temperature across the orifice, and preferentially keep the gas temperature above freezing to avoid ice formation. The equations for compressible gas flow across a nozzle are a bit different.

For this we need to introduce an entire new set of terms:

In general, equation (3) is applicable only for incompressible flows. It can be modified by introducing the expansion factor to account for the compressibility of gases.

Mass Flow,  $m$  is equal to the density ( $\rho_1$ ) times the flow or  $Q$

Then:  $m = Q * \rho = C * Y * A_2 * \sqrt{2\rho_1} * (P_1 - P_2)$  (Equation 5)

$Y$  is an expansion factor equal to 1.0 for incompressible fluids and it can be calculated for compressible gases.  $C$  is the nozzle coefficient  $A_2$  is the area of the opening,  $P_1$  is the upstream pressure  $P_2$  is the downstream pressure, and  $\rho_1$  is the upstream gas density.

Calculation of expansion factor

The expansion factor  $Y$ , which allows for the change in the density of an ideal gas as it expands isentropically, is given by Equation 6.

$$Y = \sqrt{r^{2/k} \left( \frac{k}{k-1} \right) \left( \frac{1 - r^{(k-1)/k}}{1 - r} \right) \left( \frac{1 - \beta^4}{1 - \beta^4 r^{2/k}} \right)} \quad (\text{Equation 6})$$

For values of  $\beta$  (ratio of orifice diameters to the pipe) less than 0.25  $\beta^4$  approaches 0 and the last bracketed term in the above equation approaches 1. Thus, for the large majority of orifice plate installations:

$$Y = \sqrt{r^{2/k} \left( \frac{k}{k-1} \right) \left( \frac{1 - r^{(k-1)/k}}{1 - r} \right)} \quad (\text{Equation 7})$$

$Y$  = Expansion factor, dimensionless

$r = P_2/P_1$  (Absolute pressures)

$k$  = specific heat ratio ( $c_p/c_v$ ), dimensionless, but for air it is 1.4 which is good enough for most cases, unless one really has a heavy vapor concentration in the gas.<sup>xi</sup>

Substituting equation (6) into the mass flow rate equation, and making a few substitutions using the Gas Law, we get:

$$Q = CA_2 \sqrt{\frac{2ZRT_1}{M} \left(\frac{k}{k-1}\right) \left[ \left(\frac{P_2}{P_1}\right)^{\frac{2}{k}} - \left(\frac{P_2}{P_1}\right)^{\frac{k+1}{k}} \right]} \quad (\text{Equation 8})$$

Where  $T_1$  is the initial temperature, and thus, the final equation for the non-choked (i.e., sub-sonic) flow of ideal gases through an orifice for values of  $\beta$  less than 0.25: where:

$k$  = specific heat ratio ( $c_p/c_v$ ), dimensionless

$m$  = mass flow rate at any section, kg/s  $Q_1$   $Q$  = upstream real gas flow rate, m<sup>3</sup>/s

$C$  = orifice flow coefficient, dimensionless (generally around 0.60-0.66)

$A_2$  = cross-sectional area of the orifice hole, m<sup>2</sup>

$P_1$  = upstream gas pressure, Pa with dimensions of kg/(m·s<sup>2</sup>)

$P_2$  = downstream pressure, Pa with dimensions of kg/(m·s<sup>2</sup>)

$M$  = the gas molecular mass, kg/mol (also known as the molecular weight)

$R$  = the Universal Gas Law Constant = 8.3145 J/(mol·K)

$T_1$  = absolute upstream gas temperature, K

$Z$  = the gas compressibility factor at  $P_1$  and  $T_1$  and , dimensionless—but most of the time it is 1 for air at environmental temperatures generally encountered<sup>xii</sup>.

Two final steps are required: 1) a final check of the velocity of the gas through the orifice, ( $V_{\text{Velocity through the orifice}} = Q_{\text{orifice}}/A_{\text{orifice}}$ . The gas velocity should not exceed the speed of sound<sup>xiii</sup> because that complicates the calculation by changing the effective orifice size<sup>xiv</sup>; and 2) The temperature of the expanded gas after the orifice should be checked to verify that the gas will remain above freezing. This is important for both vacuum extraction and vapor venting horizontal wells (under pressure) because there is the possibility of reducing the permeability of the formation by freezing the space around each of the individual holes if the temperature of the expanded gas gets too low. One may not have a lot of flexibility in the design because the temperature of most soils is between 50°-52° F (10°C± 2°C)

The entire program is easily arranged on an Excel spreadsheet, and the total orifice size and pressure drop and gas flow can be easily calculated.

Once the pressure drop and gas properties are identified, a drill size and hole spacing can be selected to achieve optimum distribution of the air or liquid flow.

A brief calculation of the total orifice size and appropriate area is very straight forward and easily performed. A copy of the Excel<sup>®</sup> spread sheet is shown below. The formulation is given in most chemical engineering text books or even on Wikipedia.

Calculates	$w = C_o F_r e Y S_o \sqrt{2g_c(p_a - p_b) \rho_a}$			
where:	w=	calc	0.006818	lbs/sec
Co( see pg. 105, McCabe & Smith)=	0.725	Y=	0.872848	
Fr=	1			delta P
Y=	calc			
So=	calc	2.13E-05		
gc=	32.2			
pa=	calc	16516.8		
pb=	calc	9345.6		
rhoa=	calc	0.553767		
rhob=	calc	0.276788		
Upstream Gas temperature, deg F=	100			
Downstream Gas temperature, deg F=	173.94			
Orifice Dia.=	0.0625			
Pipe Diameter	6.065			
Beta Ratio	calc	0.010305		
Molecular Weight of Gas	29			
Upstream orifice tap pressure, psig	100		57.35	psia upstream/2
Downstream orifice tap pressure, psig	50.2		80.08	Constraint # 1
Delta P across orifice	calc	49.8	-29.88	Constraint # 2
K value for gas	1.4			Constraint # 3
q, cfm=	calc	1.477992		
at	50.2			psig downstream after pressure recovery
q , scfm	5.346382			scfm
Pressure Recovery factor , Fig 2-39=	0.6			Downstream actual pressure, psig
Actual delta P	29.88		psig	70.12
Curve from McCabe & Smith, pg 108				
Velocity across orifice	1156.198			feet per second
Sonic Velocity(pg 88, McCabe & Smith)	1159.556			
Sonic velocity flowrate	11.56584			scfm
Adiabatic temperature drop	633.9417			exit degrees Rankin
Theoretical Permanent pressure loss	49.79			

The procedure is a bit of trial and error to find the right quantity and orifice size, but well worth the trouble. An installation of 12 lines of 400 feet each in the Louisiana clays successfully removed over 100 tons of chloroform and carbon tetrachloride in the ground in a period of around 6 months.

## 6 Further Reading for Information:

Most of the available information on horizontal wells has been obtained from the petroleum industry on the subject of analysis of reservoirs and horizontal wells. Recommendations for additional reading include:

Groundwater and Wells by Fletcher Driscoll (1986) Available from Amazon.com, Johnson Well Screens, and other sources.

Horizontal Well Technology by Sada Joshi, (1991),

Horizontal Wells: Focus on the Reservoir, by Timothy R. Carr and Erik P. Mason May 1, 2003  
[http://petrowiki.org/Horizontal\\_wells](http://petrowiki.org/Horizontal_wells)

Horizontal wells and directional drilling is often treated as part of reservoir engineering in the petroleum industry and additional information may be found in texts on that subject.

## Endnotes

<sup>i</sup> US Geologic Survey Professional Paper # 708, Ground-Water Hydraulics, published 1972, Available from USGS for free download.

<sup>ii</sup> For field methods of determination and sample calculations, the USEPA Method 9100 Saturated hydraulic conductivity, saturated leachate conductivity and intrinsic permeability has some useful information. The publication is available from <https://www.epa.gov/sites/production/files/2015-12/documents/9100.pdf>

<sup>iii</sup> Pay Attention to the units of viscosity and area. There is a kinematic viscosity and a dynamic viscosity. The dynamic viscosity is expressed by the symbol  $\mu$ ; The kinematic viscosity is expressed by the symbol  $\nu$  and it is the dynamic viscosity divided by the density of the fluid  $\rho$ . Generally these units are metric but occasionally English units can be used.

<sup>iv</sup> A confined aquifer has impermeable layers above and below the permeable layer. This structure is often typical of problems encountered in the petroleum industry for oil extraction.

<sup>v</sup> Johnson Well Screens: <http://johnsonwellproducts.com/product-finder.html> advertises a 6" diameter (15.24 cm) well screen with between 35 and 160 square inches (225 -1032 sq.cm) of screen opening per foot (30.48 cm) of length. This allows high flow of fluids because of the open area, and will concentrate the flow in the pipe (both suction and discharge) near the first well screen in a system)

<sup>vi</sup> The implications of the greater open area is that for the first few feet of a vacuum or pressure application, the flows will be highest in the first few feet of well screen. The flows at the furthest extent of the well screen will be very low to minimum.

<sup>vii</sup> This does not indicate that the local perforated PVC drainage pipe available from the local hardware or builder's supply should be used for distribution or collection because the openings in the pipe be too large for development of the desired flow patters.

<sup>viii</sup> Reynolds Number  $R_e$  is the ratio between inertial forces and viscous forces in a flowing fluid,

$R_e$  is calculated as 
$$R_e = \frac{VD\rho}{\mu}$$

where:  $\rho$  is the density of the fluid (SI units: kg/m<sup>3</sup>), D is a characteristic linear dimension (usually given as pipe diameter) (m),  $\mu$  is the dynamic viscosity of the fluid (Pa·s or N·s/m<sup>2</sup> or kg/m·s), and V is the velocity of the fluid within the pipe. For Reynolds numbers of less than 2000, the flow is laminar, and above 4000 it is generally considered as turbulent flow.

<sup>ix</sup> Water Resources Bulletin, Vol 28, No 3, June 1992, American Water Resources Association. Article by Duchene and McBean

<sup>x</sup> Water Supply and Wastewater Disposal, by GM Fair and GC Geyer, John Wiley and Sons, NY, pp686-689

<sup>xi</sup>  $C_v$  and  $C_p$  are tabulated on Internet sites such as: [http://www.engineeringtoolbox.com/specific-heat-capacitygases-d\\_159.html](http://www.engineeringtoolbox.com/specific-heat-capacitygases-d_159.html)

<sup>xii</sup> Compressibility factor is a measurement of the variation of a select gas versus an Ideal gas under specific temperature and pressure conditions. The subject is beyond the scope of this article.

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<sup>xiii</sup> Speed of sound is 343 m/s or approximately 1125 feet per seconds.

<sup>xiv</sup> Exceeding the speed of sound creates a mini-shockwave, and creates “choked flow” in the orifice hole, reducing the aperture.